

Operational Amplifiers

INTRODUCTION:

An operational amplifier, or **op-amp**, is a very high gain differential amplifier with high input impedance and low output impedance. Typical uses of the operational amplifier are to provide voltage amplitude changes (amplitude and polarity), oscillators, filter circuits, and many types of instrumentation circuits. An op-amp contains a number of differential amplifier stages to achieve a very high voltage gain.

Figure 10.1 shows a basic op-amp with two inputs and one output as would result using a differential amplifier input stage. Each input results in either the same or an opposite polarity (or phase) output, depending on whether the signal is applied to the plus (+) or the minus (-) input, respectively.

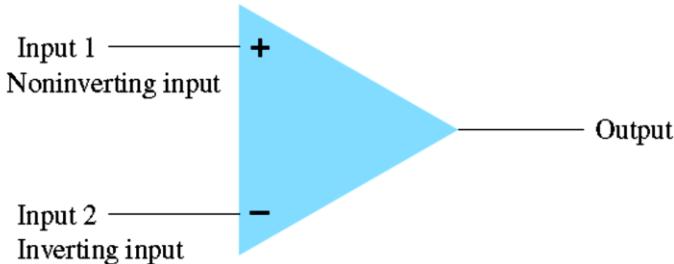
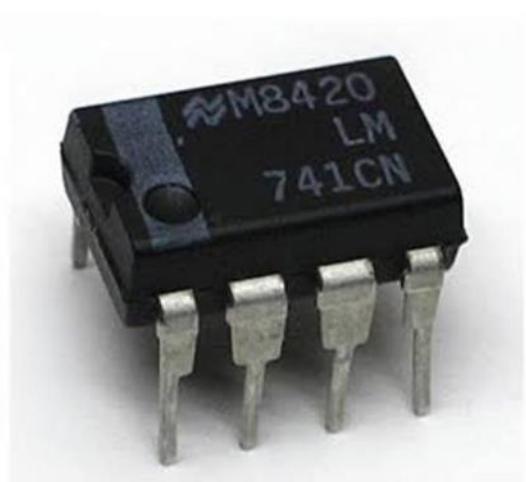


FIG. 10.1
Basic op-amp.



Single-Ended Input:

Single-ended input operation results when the input signal is connected to one input with the other input connected to ground. **Figure 10.2** shows the signals connected for this operation.

In Fig. 10.2 a, the input is applied to the plus input (with minus input at ground), which results in an output having the same polarity as the applied input signal.

Figure 10.2 b shows an input signal applied to the minus input, the output then being opposite in phase to the applied signal.

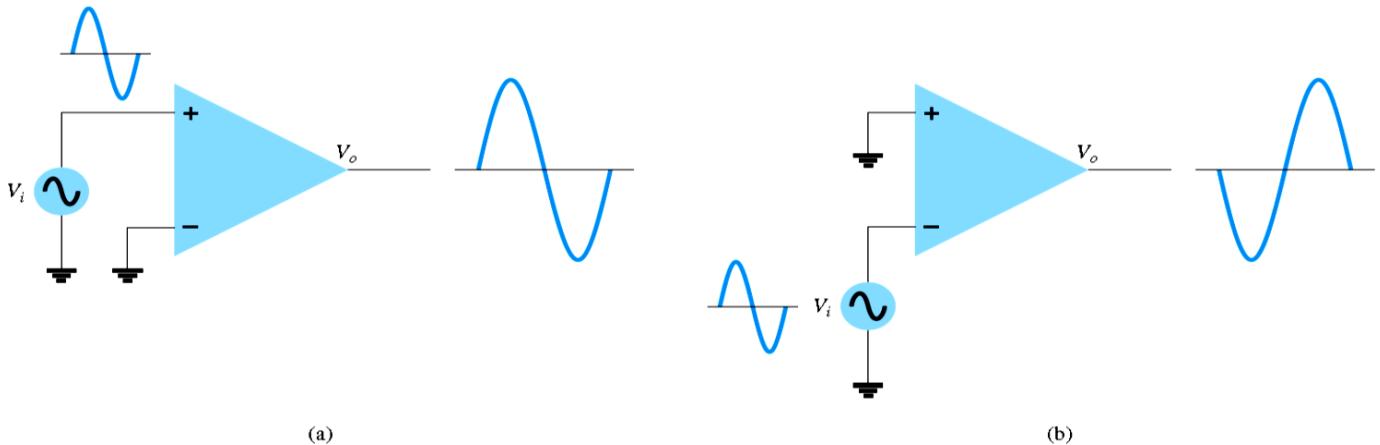


FIG. 10.2
Single-ended operation.

Double-Ended (Differential) Input:

In addition to using only one input, it is possible to apply signals at each input—this being a double-ended operation. Figure 10.3 a shows an input, V_d , applied between the two input terminals (recall that neither input is at ground), with the resulting amplified output in phase with that applied between the plus and minus inputs. Figure 10.3 b shows the same action resulting when two separate signals are applied to the inputs, the difference signal being $V_{i1} - V_{i2}$.

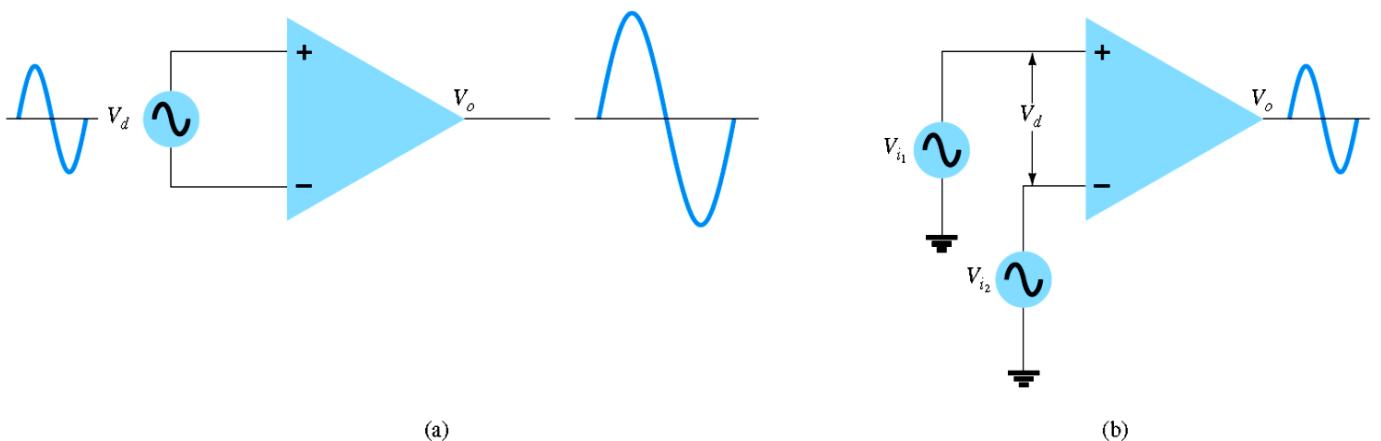
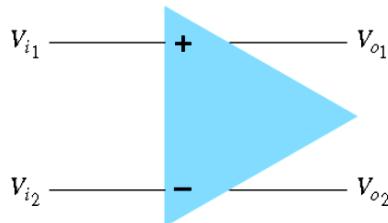


FIG. 10.3
Double-ended (differential) operation.

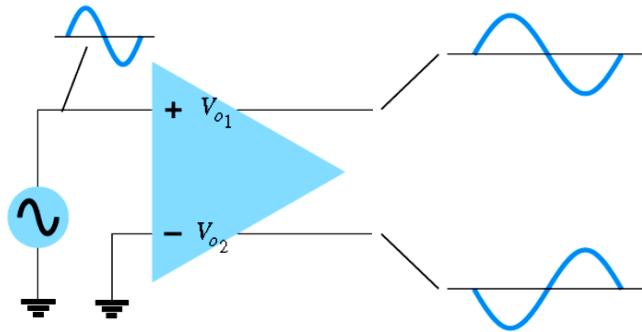
Double-Ended Output:

Whereas the operation discussed so far has a single output, the op-amp can also be operated with opposite outputs, as shown in **Fig. 10.4**. An input applied to either input will result in outputs from both output terminals, these outputs always being opposite in polarity.

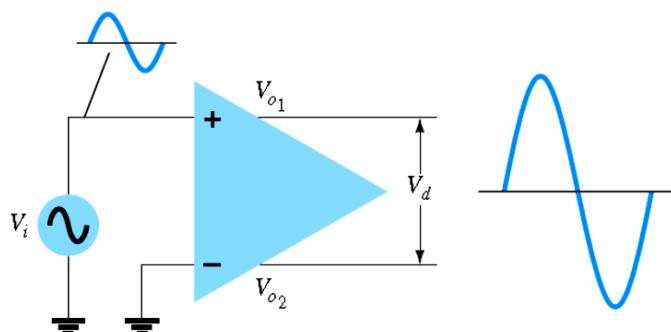
Figure 10.5 shows a single-ended input with a double-ended output. As shown, the signal applied to the plus input results in two amplified outputs of opposite polarity. **Figure 10.6** shows the same operation with a single output measured between output terminals (not with respect to ground). This difference output signal is $V_{o1} - V_{o2}$. The difference output is also referred to as a *floating signal* since neither output terminal is the ground (reference) terminal. The difference output is twice as large as either V_{o1} or V_{o2} because they are of opposite polarity and subtracting them results in twice their amplitude.

**FIG. 10.4**

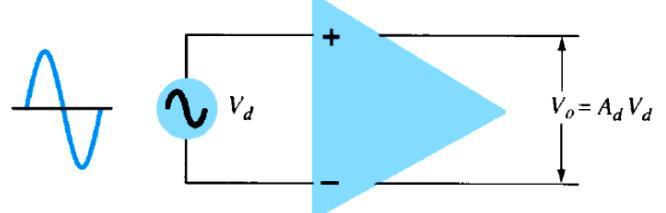
Double-ended input with double-ended output.

**FIG. 10.5**

Single-ended input with double-ended output.

**FIG. 10.6**

Differential-output.

**FIG. 10.7**

Differential-input, differential-output operation.

[e.g., $10\text{ V} - (-10\text{ V}) = 20\text{ V}$]. Figure 10.7 shows a differential input, differential output operation. The input is applied between the two input terminals and the output taken from between the two output terminals. This is fully differential operation.

Common-Mode Operation:

When the same input signals are applied to both inputs, common-mode operation results, as shown in Fig. 10.8. Ideally, the two inputs are equally amplified, and since they result in opposite-polarity signals at the output, these signals cancel, resulting in 0-V output. Practically, a small output signal will result.

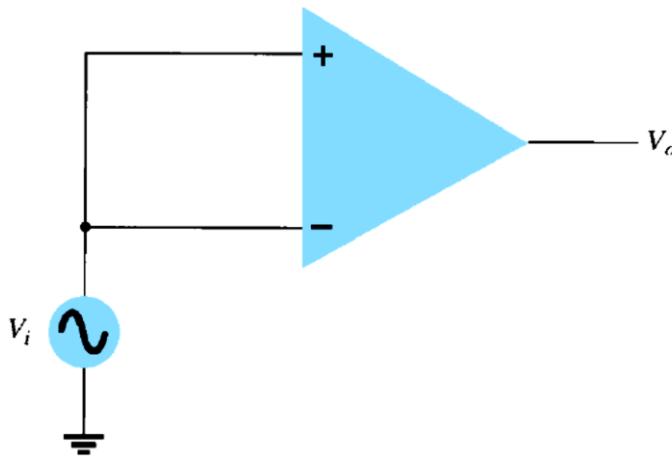


FIG. 10.8
Common-mode operation.

Common-Mode Rejection:

A significant feature of a differential connection is that the signals that are opposite at the inputs are highly amplified, whereas those that are common to the two inputs are only slightly amplified—the overall operation being to amplify the difference signal while rejecting the common signal at the two inputs. Since noise (any unwanted input signal) is generally common to both inputs, the differential connection tends to provide attenuation of this unwanted input while providing an amplified output of the difference signal applied to the inputs. This operating feature is referred to as common-mode rejection.

DIFFERENTIAL AMPLIFIER CIRCUIT :

The differential amplifier circuit is an extremely popular connection used in IC units. This connection can be described by considering the basic differential amplifier shown in **Fig. 10.9**. Notice that the circuit has two separate inputs and two separate outputs, and that the emitters are connected together. Whereas many differential amplifier circuits use two separate voltage supplies, the circuit can also operate using a single supply.

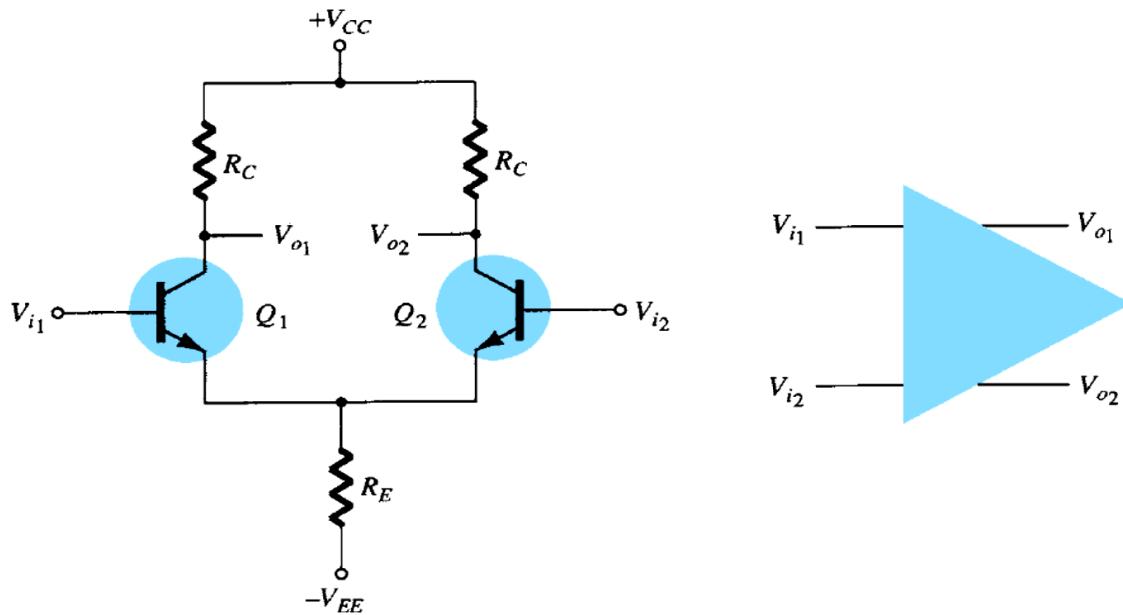


FIG. 10.9
Basic differential amplifier circuit.

A number of input signal combinations are possible:

If an input signal is applied to either input with the other input connected to ground, the operation is referred to as “single-ended.”

If two opposite-polarity input signals are applied, the operation is referred to as “double-ended.”

If the same input is applied to both inputs, the operation is called “common-mode.”

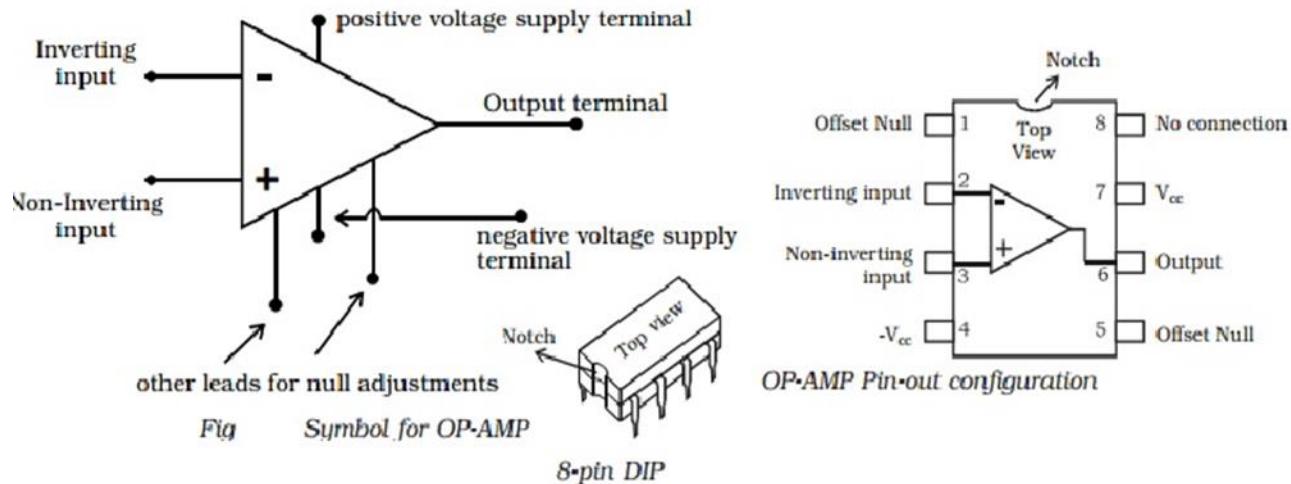
In single-ended operation, a single input signal is applied. However, due to the common emitter connection, the input signal operates both transistors, resulting in output from both collectors.

In double-ended operation, two input signals are applied, the difference of the inputs resulting in outputs from *both* collectors due to the difference of the signals applied to both inputs.

In common-mode operation, the common input signal results in opposite signals at each collector, these signals canceling, so that the resulting output signal is zero. As a practical matter, the opposite signals do not completely cancel, and a small signal results.

The main feature of the differential amplifier is the very large gain when opposite signals are applied to the inputs as compared to the very small gain resulting from common inputs.

The ratio of this difference gain to the common gain is called *common-mode rejection*.



OP-AMP BASICS:

An operational amplifier is a very high gain amplifier having very high input impedance (typically a few mega ohms) and low output impedance (less than 100Ω) . The basic circuit is made using a difference amplifier having two inputs (plus and minus) and at least one output. Figure 10.29 shows a basic op-amp unit. As discussed earlier, the plus (+) input produces an output that is in phase with the signal applied, whereas an input to the minus (-) input results in an opposite-polarity output. The ac equivalent circuit of the op-amp is shown in Fig. 10.30 a. As shown, the input signal applied between input terminals sees an input impedance R_i that is typically very high. The output voltage is shown to be the amplifier gain times the input signal taken through an output impedance R_o , which is typically very low. An ideal op-amp circuit, as shown in Fig. 10.30 b, would have infinite input impedance, zero output impedance, and infinite voltage gain.

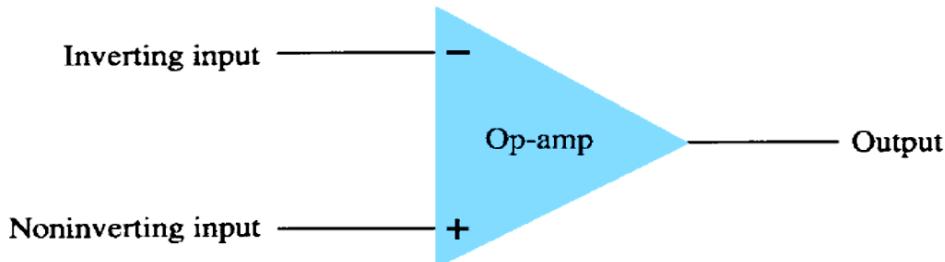


FIG. 10.29
Basic op-amp.

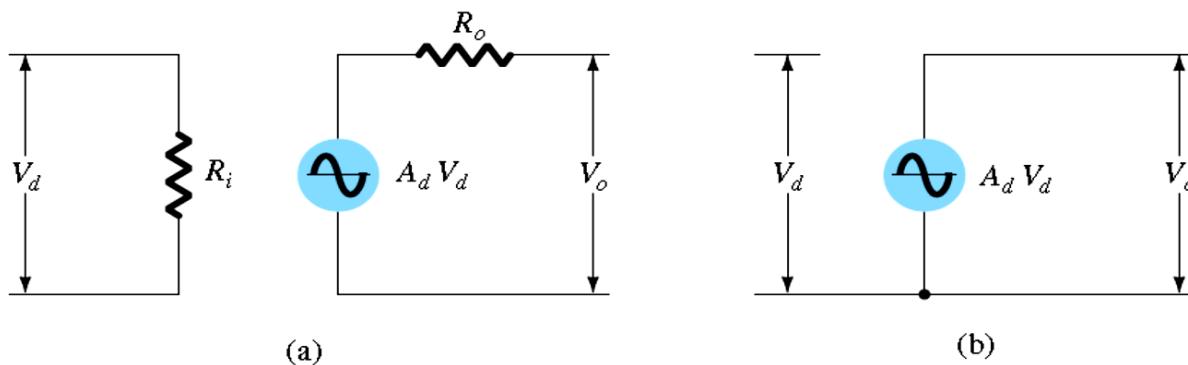


FIG. 10.30
AC equivalent of op-amp circuit: (a) practical; (b) ideal.

Basic Op-Amp:

The basic circuit connection using an op-amp is shown in Fig. 10.31. The circuit shown provides operation as a constant-gain multiplier. An input signal V_1 is applied through resistor R_1 to the minus input. The output is then connected back to the same minus input through resistor R_f . The plus input is connected to ground. Since the signal V_1 is essentially applied to the minus input, the resulting output is opposite in phase to the input signal.

Figure 10.32 a shows the op-amp replaced by its ac equivalent circuit. If we use the ideal op-amp equivalent circuit, replacing R_i by an infinite resistance and R_o by a zero resistance, the ac equivalent circuit is that shown in Fig. 10.32 b. The circuit is then redrawn, as shown in Fig. 10.32 c, from which circuit analysis is carried out.

Using superposition, we can solve for the voltage V_1 in terms of the components due to each of the sources. For source V_1 only ($-A_v V_i$ set to zero),

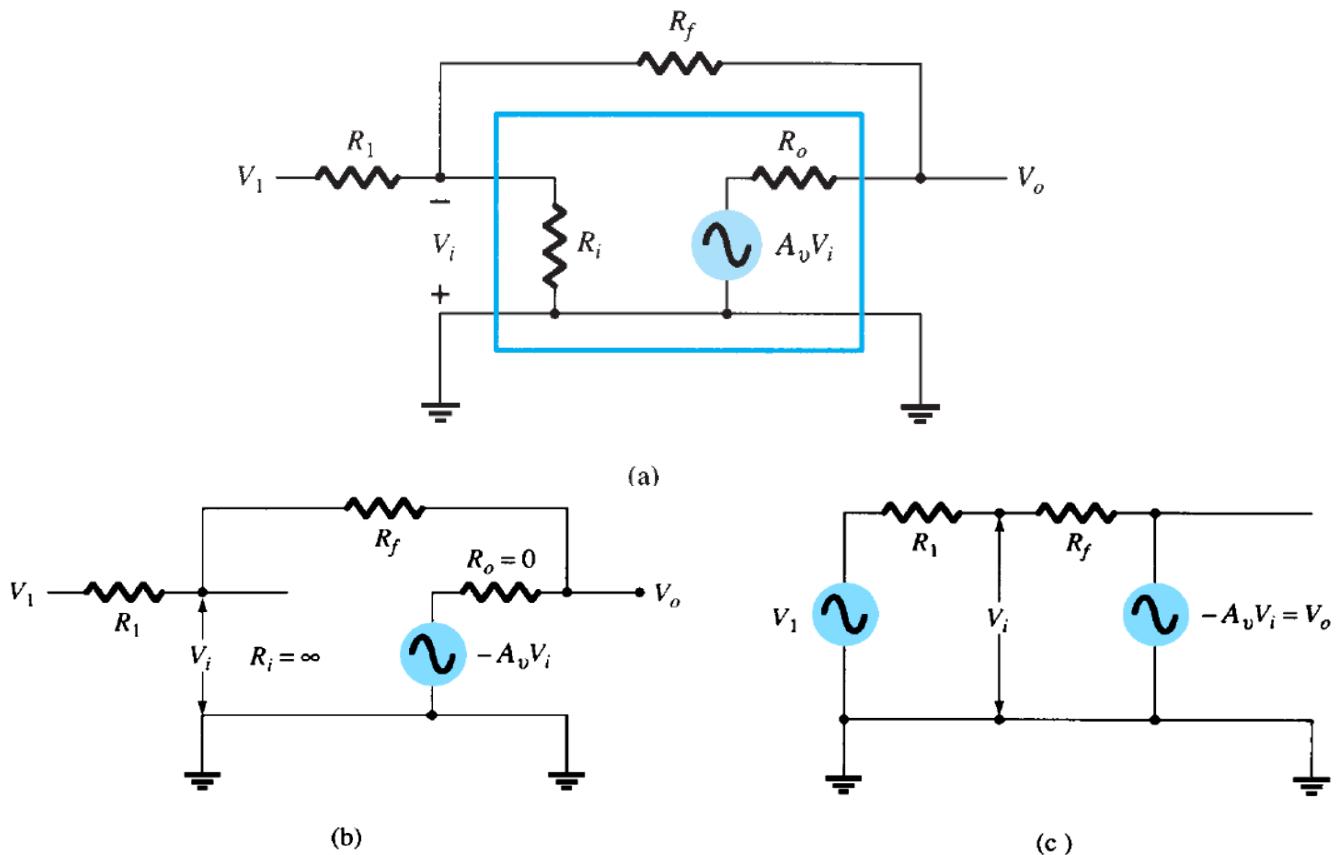


FIG. 10.32

Operation of op-amp as constant-gain multiplier: (a) op-amp ac equivalent circuit; (b) ideal op-amp equivalent circuit; (c) redrawn equivalent circuit.

$$V_{i_1} = \frac{R_f}{R_1 + R_f} V_1$$

For source $-A_v V_i$ only (V_1 set to zero),

$$V_{i_2} = \frac{R_1}{R_1 + R_f} (-A_v V_i)$$

The total voltage V_i is then

$$V_i = V_{i_1} + V_{i_2} = \frac{R_f}{R_1 + R_f} V_1 + \frac{R_1}{R_1 + R_f} (-A_v V_i)$$

which can be solved for V_i as

$$V_i = \frac{R_f}{R_f + (1 + A_v)R_1} V_1 \quad (10.7)$$

If $A_v \gg 1$ and $A_v R_1 \gg R_f$, as is usually true, then

$$V_i = \frac{R_f}{A_v R_1} V_1$$

Solving for V_o/V_i , we get

$$\frac{V_o}{V_i} = \frac{-A_v V_i}{V_i} = \frac{-A_v}{V_i} \frac{R_f V_1}{A_v R_1} = -\frac{R_f}{R_1} \frac{V_1}{V_i}$$

so that

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1} \quad (10.8)$$

The result in Eq. (10.8) shows that the ratio of overall output to input voltage is dependent only on the values of resistors R_1 and R_f —provided that A_v is very large.

Unity Gain

If $R_f = R_1$, the gain is

$$\text{Voltage gain} = -\frac{R_f}{R_1} = -1$$

so that the circuit provides a unity voltage gain with 180° phase inversion. If R_f is exactly R_1 , the voltage gain is exactly 1.

PRACTICAL OP-AMP CIRCUITS :

Inverting Amplifier

The most widely used constant-gain amplifier circuit is the inverting amplifier, as shown in Fig. 10.34. The output is obtained by multiplying the input by a fixed or constant gain, set by the input resistor (R_1) and feedback resistor (R_f)—this output also being inverted from the input. Using Eq. (10.8), we can write

$$V_o = -\frac{R_f}{R_1} V_1$$

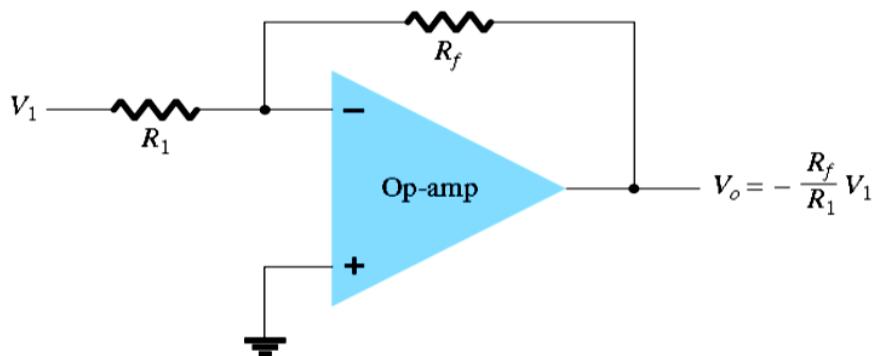


FIG. 10.34
Inverting constant-gain multiplier.

EXAMPLE 10.5 If the circuit of Fig. 10.34 has $R_1 = 100 \text{ k}\Omega$ and $R_f = 500 \text{ k}\Omega$, what output voltage results for an input of $V_1 = 2 \text{ V}$?

Solution:

$$\text{Eq. (10.8): } V_o = -\frac{R_f}{R_1} V_1 = -\frac{500 \text{ k}\Omega}{100 \text{ k}\Omega} (2 \text{ V}) = -10 \text{ V}$$

Noninverting Amplifier

The connection of Fig. 10.35a shows an op-amp circuit that works as a noninverting amplifier or constant-gain multiplier. It should be noted that the inverting amplifier connection is more widely used because it has better frequency stability (discussed later). To determine the voltage gain of the circuit, we can use the equivalent representation shown in Fig. 10.35b. Note that the voltage across R_1 is V_1 since $V_i \approx 0 \text{ V}$. This must be equal to the output voltage, through a voltage divider of R_1 and R_f , so that

$$V_1 = \frac{R_1}{R_1 + R_f} V_o$$

which results in

$$\frac{V_o}{V_1} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1} \quad (10.9)$$

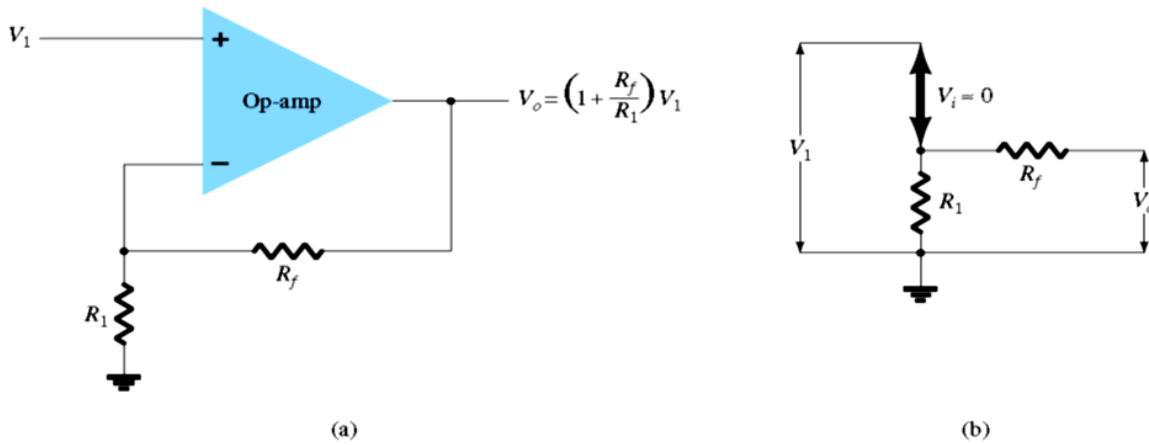


FIG. 10.35
Noninverting constant-gain multiplier.

EXAMPLE 10.6 Calculate the output voltage of a noninverting amplifier (as in Fig. 10.35) for values of $V_1 = 2$ V, $R_f = 500$ k Ω , and $R_1 = 100$ k Ω .

Solution:

$$\text{Eq. (10.9): } V_o = \left(1 + \frac{R_f}{R_1}\right)V_1 = \left(1 + \frac{500 \text{ k}\Omega}{100 \text{ k}\Omega}\right)(2 \text{ V}) = 6(2 \text{ V}) = +12 \text{ V}$$

Unity Follower

The unity-follower circuit, as shown in Fig. 10.36a, provides a gain of unity (1) with no polarity or phase reversal. From the equivalent circuit (see Fig. 10.36b) it is clear that

$$V_o = V_1 \quad (10.10)$$

and that the output is the same polarity and magnitude as the input. The circuit operates like an emitter- or source-follower circuit except that the gain is exactly unity.

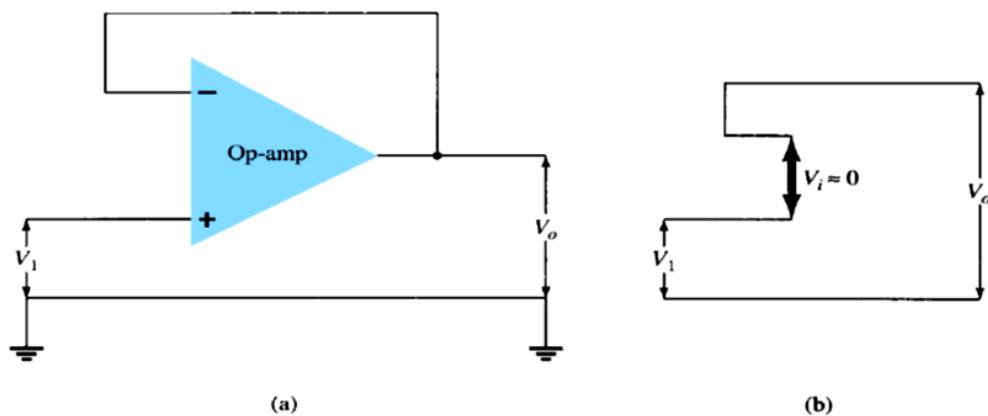


FIG. 10.36
(a) Unity follower; (b) virtual-ground equivalent circuit.

Summing Amplifier

Probably the most used of the op-amp circuits is the summing amplifier circuit shown in Fig. 10.37a. The circuit shows a three-input summing amplifier circuit, which provides a means of algebraically summing (adding) three voltages, each multiplied by a constant-gain

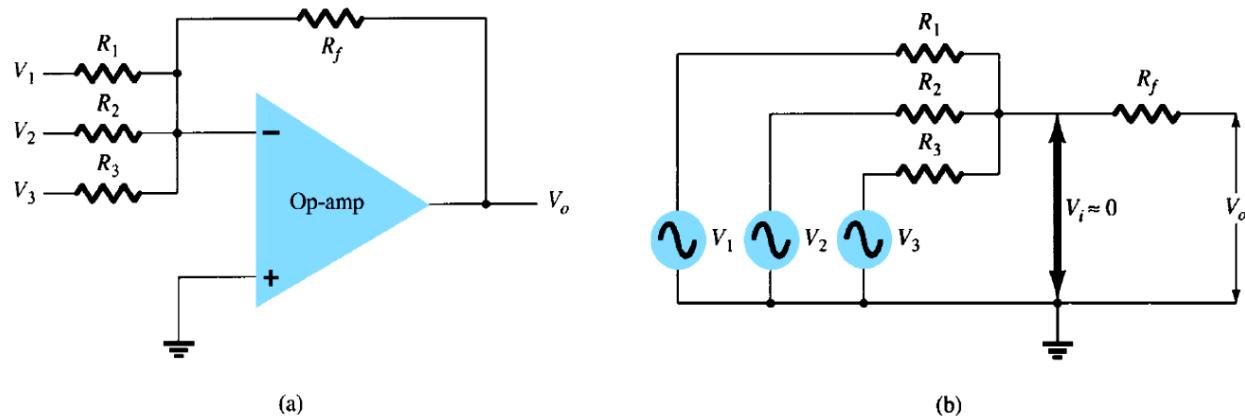


FIG. 10.37
(a) Summing amplifier; (b) virtual-ground equivalent circuit.

factor. Using the equivalent representation shown in Fig. 10.37b, we can express the output voltage in terms of the inputs as

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right) \quad (10.11)$$

In other words, each input adds a voltage to the output multiplied by its separate constant-gain multiplier. If more inputs are used, they each add an additional component to the output.

EXAMPLE 10.7 Calculate the output voltage of an op-amp summing amplifier for the following sets of voltages and resistors. Use $R_f = 1 \text{ M}\Omega$ in all cases.

- $V_1 = +1 \text{ V}$, $V_2 = +2 \text{ V}$, $V_3 = +3 \text{ V}$, $R_1 = 500 \text{ k}\Omega$, $R_2 = 1 \text{ M}\Omega$, $R_3 = 1 \text{ M}\Omega$.
- $V_1 = -2 \text{ V}$, $V_2 = +3 \text{ V}$, $V_3 = +1 \text{ V}$, $R_1 = 200 \text{ k}\Omega$, $R_2 = 500 \text{ k}\Omega$, $R_3 = 1 \text{ M}\Omega$.

Solution: Using Eq. (10.11), we obtain

$$\begin{aligned} \text{a. } V_o &= -\left[\frac{1000 \text{ k}\Omega}{500 \text{ k}\Omega}(+1 \text{ V}) + \frac{1000 \text{ k}\Omega}{1000 \text{ k}\Omega}(+2 \text{ V}) + \frac{1000 \text{ k}\Omega}{1000 \text{ k}\Omega}(+3 \text{ V})\right] \\ &= -[2(1 \text{ V}) + 1(2 \text{ V}) + 1(3 \text{ V})] = -7 \text{ V} \\ \text{b. } V_o &= -\left[\frac{1000 \text{ k}\Omega}{200 \text{ k}\Omega}(-2 \text{ V}) + \frac{1000 \text{ k}\Omega}{500 \text{ k}\Omega}(+3 \text{ V}) + \frac{1000 \text{ k}\Omega}{1000 \text{ k}\Omega}(+1 \text{ V})\right] \\ &= -[5(-2 \text{ V}) + 2(3 \text{ V}) + 1(1 \text{ V})] = +3 \text{ V} \end{aligned}$$

5- Subtractor

The function of a subtractor is to provide an output proportional to the difference of two input signals. As shown in Fig. 7. The inputs are applying at the inverting and noninverting terminals.

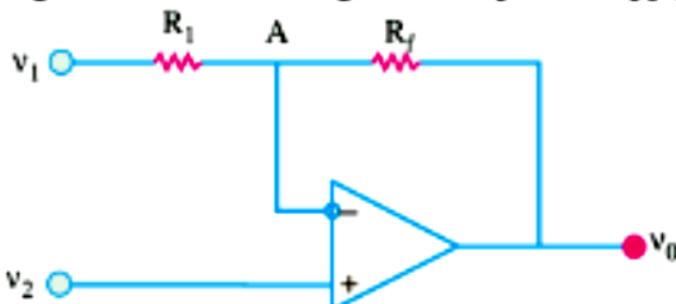


Fig. 7

Calculations

According to Superposition theorem;

$$v_0 = v_0' + v_0''$$

where v_0' is the output produced by v_1 and v_0'' is that produced by v_2 .

$$\text{Now } v_0' = -\frac{R_f}{R_1} \cdot v_1 \text{ (see inverting amplifier equation)}$$

$$v_0'' = \left(1 + \frac{R_f}{R_1}\right) v_2 \text{ (see noninverting amplifier equation)}$$

If $R_f \gg R_1$ and $R_f/R_1 \gg 1$, hence

$$v_0 \equiv \frac{R_f}{R_1} (v_2 - v_1)$$

Further, If $R_f = R_1$, then

$v_0 = (v_2 - v_1)$ = difference of the two input voltages

Example: Find the output voltages of an OP-AMP inverting adder for the following sets of input voltages and resistors. In all cases, $R_f = 1 \text{ M}\Omega$.

$$v_1 = -3 \text{ V}, v_2 = +3 \text{ V}, v_3 = +2 \text{ V}; R_1 = 250 \text{ K}\Omega, R_2 = 500 \text{ K}\Omega, R_3 = 1 \text{ M} \Omega \text{ (ans. } V_0 = 4 \text{ V)}$$

Example: Design an OP-AMP circuit that will produce an output equal to $-(4 v_1 + v_2 + 0.1 v_3)$.

Write an expression for the output and sketch its output waveform when $v_1 = 2 \sin \omega t$, $v_2 = -5 \text{ V}$ dc and $v_3 = -100 \text{ Vdc}$.

Solution:

$$v_0 = -\left[\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right] \quad \dots(1)$$

$$v_0 = -(4 v_1 + v_2 + 0.1 v_3) \quad \dots(2)$$

Comparing equations (1) and (2), we find,

$$\frac{R_f}{R_1} = 4, \frac{R_f}{R_2} = 1, \frac{R_f}{R_3} = 0.1$$

Therefore if we assume $R_f = 100 \text{ K}$, then $R_1 = 25 \text{ K}$, $R_2 = 100 \text{ K}$ and $R_3 = 10 \text{ K}$. With these values of R_1 , R_2 and R_3 , the OP-AMP circuit is as shown in Fig. 8 (a).

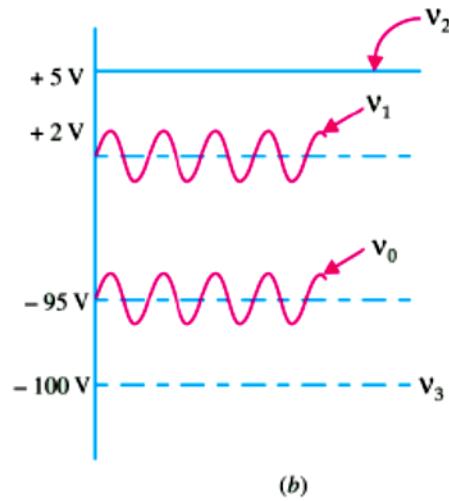
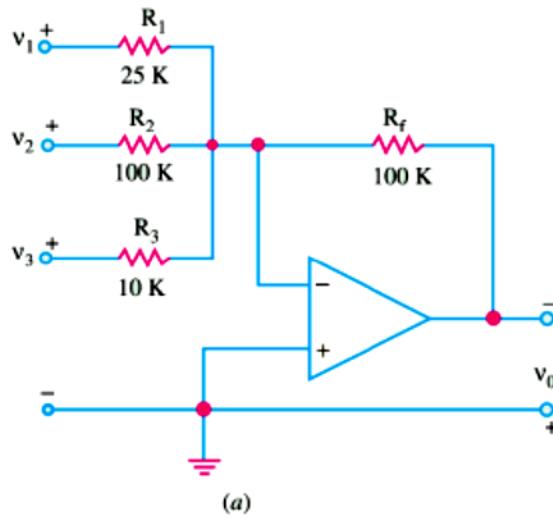


Fig. 8

With the given values of $v_1 = 2 \sin \omega t$, $v_2 = +5 \text{ V}$, $v_3 = -100 \text{ V}$ dc, the output voltage, $v_0 = 2 \sin \omega t + 5 - 100 = 2 \sin \omega t - 95 \text{ V}$. The waveform of the output voltage is sketched as shown in Fig. 8 (b).

6- Integrator

The function of an integrator is to provide an output voltage which is proportional to the integral of the input voltage. A simple example of integration is shown in Fig. 9, where input is dc level and its integral is a *linearly-increasing ramp output*. The actual integration circuit is similar to the inverting circuit except that the feedback component is a capacitor C instead of a resistor R_f .

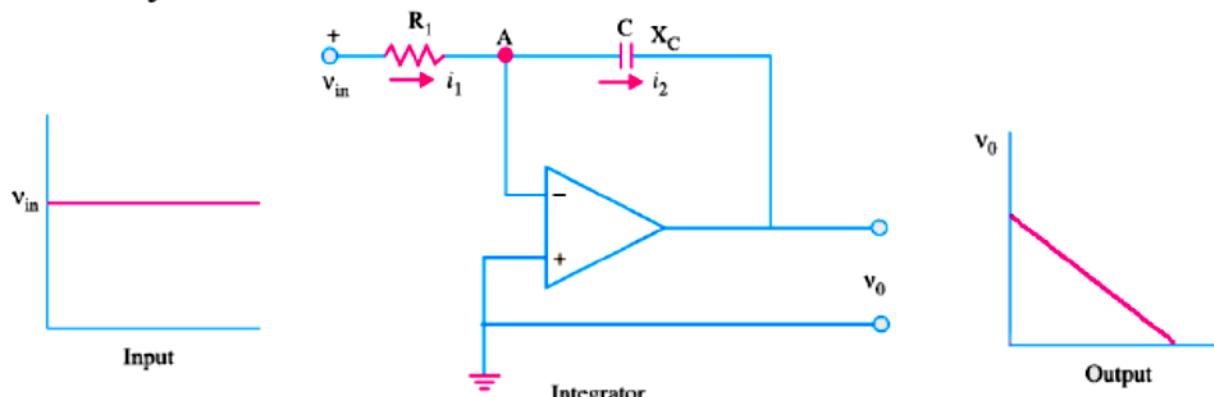


Fig. 9

Calculations

As before, If the op amp is ideal, point A will be treated as virtual ground, and v_{in} appears across R_1 . Thus

$$i_1 = \frac{v_{in}}{R_1}$$

$$i_2 = i_c = c \frac{dv_c}{dt} = -c \frac{dv_o}{dt} \quad (\text{since } v_c = -v_o)$$

But, with negligible current into the op amp, the current through R_1 = current flow through C .
Then

$$\frac{v_{in}}{R_1} = -c \frac{dv_o}{dt} \Rightarrow dv_o = -\frac{1}{R_1 C} v_{in} dt \Rightarrow v_o = -\frac{1}{R_1 C} \int v_{in} dt$$

It is seen from above that output (left-hand side) is an integral of the input, with an inversion and a scale factor of $1/CR_1$. This ability to integrate a given signal enables an analog computer solve differential equations and to set up a wide variety of electrical circuit analogs of physical system operation.

Note: we can integrate more than one input as shown below in Fig. 10. With multiple inputs, the output is given by

$$v_o(t) = - \left[K_1 \int v_1(t) dt + K_2 \int v_2(t) dt + K_3 \int v_3(t) dt \right]$$

where $K_1 = \frac{1}{CR_1}$, $K_2 = \frac{1}{CR_2}$ and $K_3 = \frac{1}{CR_3}$

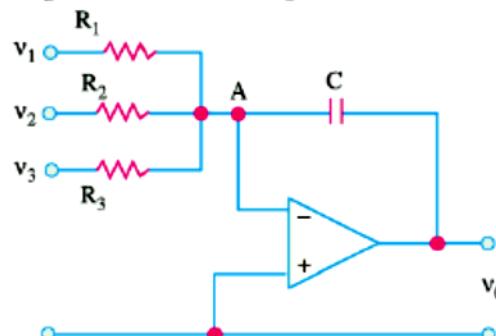


Fig. 10

Example: A 5mV, 1-kHz sinusoidal signal is applied to the input of an OP-AMP integrator, for which $R = 100$ K and $C = 1 \mu\text{F}$. Find the output voltage.

Solution:

$$-\frac{1}{CR} = \frac{1}{10^5 \times 10^{-6}} = -10$$

The equation for the sinusoidal voltage is

$$v_1 = 5 \sin 2\pi ft = 5 \sin 2000 \pi t$$

Obviously, it has been assumed that at $t = 0$, $v_1 = 0$

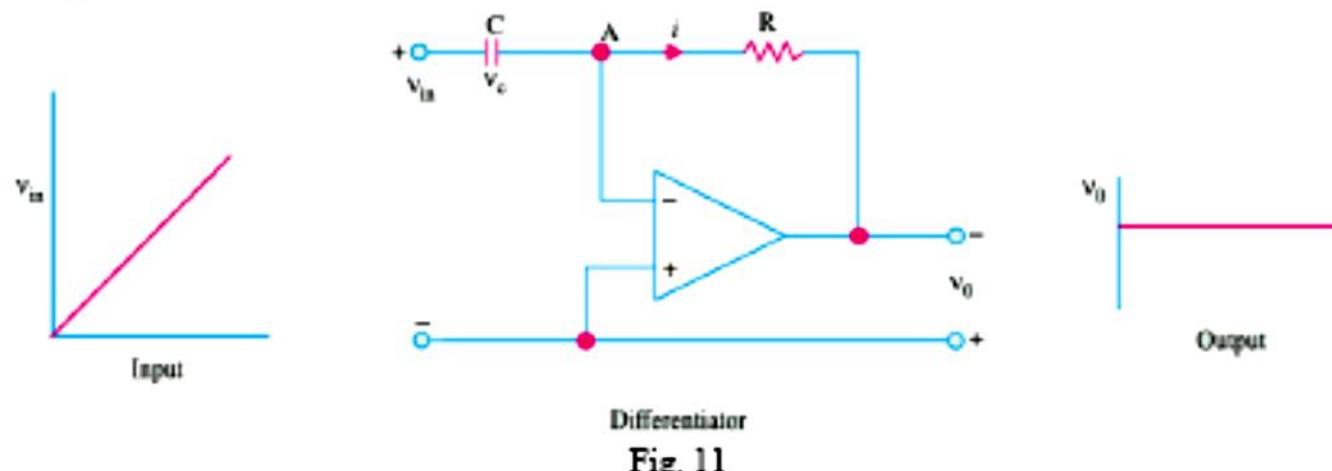
$$\begin{aligned} v_0(t) &= -10 \int_0^t 5 \sin 2000 \pi t = -50 \left| \frac{-\cos 2000 \pi t}{2000} \right|_0^t \\ &= -\frac{1}{40 \pi} (\cos 2000 \pi t - 1) \end{aligned}$$

7- Differentiator

Its function is to provide an output voltage which is proportional to the rate of the change of the input voltage. It is an inverse mathematical operation to that of an integrator. As shown in

Fig. 11, when we feed a differentiator with linearly-increasing ramp input, we get a constant dc output.

Differentiator circuit can be obtained by interchanging the resistor and capacitor of the integrator circuit.



Calculation:

The expression for the output signal of the inverting differentiator amplifier assuming the op amp is ideal can be derived as follows:

Taking point *A* as virtual ground, consequently, v_{in} appears across capacitor ($v_{in} = v_c$)

$$i = C \frac{dv_c}{dt} = -C \frac{dv_{in}}{dt}$$

Taking point *A* as virtual ground

$$v_0 = -iR = -\left(C \cdot \frac{dv_c}{dt}\right)R = -CR \cdot \frac{dv_c}{dt}$$

Example 68.6. The input to the differentiator circuit of Fig. 68.17 is a sinusoidal voltage of peak value of 5 mV and frequency 1 kHz. Find out the output if $R = 1000 \text{ k}\Omega$ and $C = 1 \mu\text{F}$.

Solution. The equation of the input voltage is

$$v_1 = 5 \sin 2\pi \times 1000 t = 5 \sin 2000 \pi t \text{ mV}$$

scale factor $= CR = 10^{-6} \times 10^5 = 0.1$

$$v_0 = 0.1 \frac{d}{dt} (5 \sin 2000 \pi t) = (0.5 \times 2000 \pi) \cos 2000 \pi t = 1000 \pi \cos 2000 \pi t \text{ mV}$$

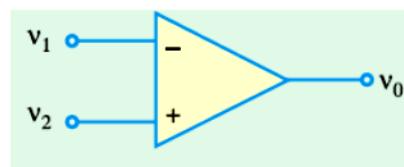


Fig. 68.18

As seen, output is a cosinusoidal voltage of frequency 1 kHz and peak value $1000 \pi \text{ mV}$.