① $0 < P(X) < 1$

② $\sum P(x_i) = 1$

س/

$f(x) = \frac{1}{6}$, $x = 1, 2, 3, 4, 5, 6$

is $f(x)$ p.m.f ?

الحل

$$P(X=1) = P(X=2) = P(X=3) = P(X=4)$$

$$= P(X=5) = P(X=6)$$

$$= \frac{1}{6}$$

دالة الكثافة الاحتمالية (P.D.F)

مستمر $0 < x < 4$

شروط

① $0 \leq f(x) \leq 1$

② $\int_{-\infty}^{\infty} f(x)\, dx = 1$

③ $(a < X < b) = \int_{a}^{b} f(x)\, dx$

④ $P(X = c) = 0$

1س

$$f(x) = e^{-x} \quad , \; 0 < x < \infty$$

is $f(x)$ p.d.f ?

① $0 \leqslant f(x) \leqslant 1$

② $\int_{-\infty}^{\infty} f(x)\, dx = \int_{0}^{\infty} e^{-x}\, dx$

$$= -1 \, e^{-x} \Big]_0^{\infty}$$

$$= -(e^{-\infty} - e^{0})$$

$$= -(0 - 1)$$

$$= 1$$

التوقع للمتقطع $E(X)$

$$E(X) = \sum x\, P(X_i)$$

$$E(X^2) = \sum x^2\, P(X_i)$$

التباين للمتقطع $\sigma^2(X)$

$$\sigma_X^2 = E(X^2) - [E(X)]^2$$

5)

X	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E(X) = \sum X_i P(X_i)$$

$$= (0)\left(\frac{1}{8}\right) + (1)\left(\frac{3}{8}\right) + (2)\left(\frac{3}{8}\right) + (3)\left(\frac{1}{8}\right)$$

$$= \frac{12}{8}$$

$$E(X^2) = \sum X_i^2 P(X_i)$$

$$= (0)^2\left(\frac{1}{8}\right) + (1)^2\left(\frac{3}{8}\right) + \cdots$$

$$= \frac{24}{8}$$

$$E(X^2) = \sum X_i^2 P(X_i)$$

$$= (0)^2\left(\frac{1}{8}\right) + (1)^2\left(\frac{3}{8}\right) + \cdots$$

$$= \frac{24}{8}$$

$$\sigma_X^2 = E(X^2) - \left[E(X)\right]^2$$

$$= \frac{24}{8} - \left[\frac{12}{8}\right]^2 = \frac{6}{8}$$

التوقع $E(X)$

المستمر

$$E(X) = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$$E(X^2) = \int_{-\infty}^{\infty} X^2 \, f(x) \, dx$$

التباين σ^2

للمستمر

$$\sigma_X^2 = E(X^2) - \left[E(X)\right]^2$$

مثلا

$$f(x) = 3x^2 \quad , \quad \underbrace{0 < x < 1}_{\text{مستمر}}$$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^1 x \cdot 3x^2\, dx$$

$$= 3\int_0^1 x^3\, dx = \frac{3}{4} x^4 \Big|_0^1$$

$$= \frac{3}{4}(1)^4 - \frac{3}{4}(0)^4 = \frac{3}{4}$$

$$= \int_0^1 x \cdot 3x^2 \, dx$$

$$= 3 \int_0^1 x^3 \, dx = \frac{3}{4} x^4 \Big|_0^1$$

$$= \frac{3}{4}(1)^4 - \frac{3}{4}(0)^4 = \frac{3}{4}$$

$$E(x^2) = \int_0^1 x^2 \cdot 3x^2 \, dx$$

$$= 3 \int_0^1 x^4 \, dx = \frac{3}{5} x^5 \Big|_0^1$$

$$= \frac{3}{5}(1)^5 - \frac{3}{5}(0)^5 = \frac{3}{5}$$

$$= 3\int_0^1 x^4\,dx = \frac{3}{5}x^5\Big|_0^1$$

$$= \frac{3}{5}(1)^5 - \frac{3}{5}(0)^5 = \frac{3}{5}$$

$$\sigma_x^2 = E(X^2) - \left[E(X)\right]^2$$

$$= \frac{3}{5} - \left(\frac{3}{5}\right)^2 = \frac{3}{80}$$