Chapter one

"Random Variables"

1– Th concept of Random variables مفهوم المتغيرات العشوائية

Definitions:

Given a random experiment with sample space S, A random variable (r.v.) X is a function which assigns to each element $w \in S$ a real number $X(w)$ in the set E .

Ex) a _ Suppose that an experiment consists of tossing two fair coins :

SoL) $S = \{HH, HT, TH, TT\}$ ____ random experiment

Random variables
- X : تمثل عدد مرات ظهور (H)
  - X= 0 , 1 , 2
- Y : تمثل عدد مرات ظهور (T)
  - Y= 0 , 1 , 2

$P(X = x)$

$P(X = 0) = \frac{1}{4}$

$P(X = 1) = \frac{2}{4}$

$P(X = 2) = \frac{1}{4}$

$\longrightarrow \sum P(X = x_i) = 1$

X Values

$HH \rightarrow 2$

$HT \rightarrow 1$

$TH \rightarrow 1$

$TT \rightarrow 0$

c _ Suppose that an experiment consists of tossing a pair of unbiased dice , X be a random variable indicating the sum of the two faces :-

Sol/

X =	2	3	4	5	6	7	8	9	10	11	12
	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)	
			(3,1)	(3,2)	(3,3)	(3,4)	(4,4)	(5,4)	(6,4)		
				(4,1)	(4,2)	(4,3)	(5,3)	(6,3)			
					(5,1)	(5,2)	(6,2)				
						(6,1)					

$$P(X=x) = \frac{1}{36}, \frac{2}{36}, \frac{3}{36}, \frac{4}{36}, \frac{5}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36}$$

X	2	3	4	5	6	7	8	9	10	11	12
P (X=x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

And $\sum P(X = x_i) = 1$

ويمكن تمثيل (r.v.) وفق دوال وعلى النحو الاتي :-

$$P(X) \begin{cases} 0 & x = 0 \\ \frac{X-1}{36} & x = 2,3,4,5,6,7 \\ \frac{13-X}{36} & x = 8,9,10,11,12 \\ 1 & \sum P(X) = 1 \end{cases}$$

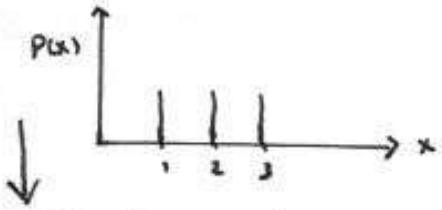
ويمكن تمثيل دالة خطية عند رمي زار مرة واحدة

$S = \{1,2,3,4,5,6\}$

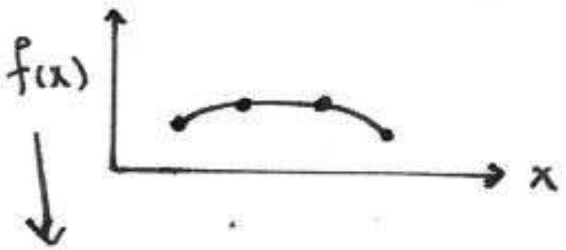
$$P(X) = \begin{cases} \frac{1}{6} & x = 1,2,3,\ldots,6 \\ 0 & \text{other wise} \end{cases}$$

Random Variable (r.v.)

متقطع	مستمر
discretes (r.v.)	Continuous (r.v.)
$x = 0, 1, 2, 3, 4, \ldots$	$a < x < b$
Probability mass function	Probability density function
دالة كتلة احتمالية	دالة كثافة احتمالية
(P.m.f.) رمزها	(P.d.f.) رمزها
حكارل $P(x)$ يعني احتمال	حكارل $f(x)$



قيم الدالة



1- $P(X) \geq 0$; $0 \leq P(X_i) \leq 1$; $i = 1, 2, 3, \ldots$

2- $\sum_{i=1}^{n} P(X_i) = 1$

1- $f(x) \geq 0$; $0 \leq f(x) \leq 1$

2- $\int_{-\infty}^{\infty} f(x)\,dx = 1$

دالة الاحتمال التراكمية

Cumulative Probability function

$$F(x) = \sum_{X_i < x} P(X_i)$$

$$F(X) = \int_{-\infty}^{x} f(x)\,dx$$

2– Distribution Function (D.F.) دالة التوزيع

Definitions:

If X is a random variable (r.v.) defined on the sample space S, the cumulative distribution function (c. d. f.) or cumulative probability function (c. p. f.) of a discrete random variable, denoted by $F(x)$ & is defined by :-

$$F(x) = P(X \leq x) \quad \text{for any real number}$$

$$= \sum P(x_i)$$

Properties :

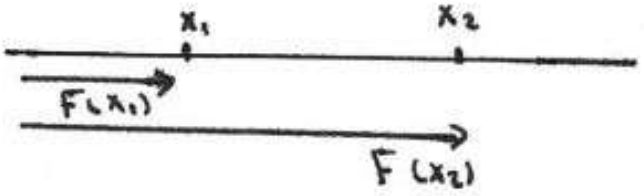
The distribution function $F(x)$ has following properties :-

1_ $F(\infty) = \lim_{x \to \infty} F(x) = 1$

2_ $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$

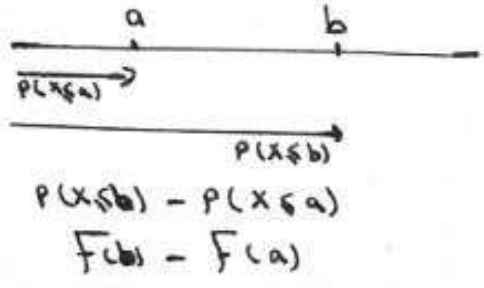
$$0 \leq F(x) \leq 1$$

3_ $F(x_1) \leq F(x_2) \quad ; \quad if \ x_1 \leq x_2$



4_ $\boldsymbol{P(a < x \leq b) = F(b) - F(a)} \quad for\ all \quad a < b$

$$\boldsymbol{P(x > b) = 1 - F(b)}$$



Ex)

$$\boldsymbol{P(x) = \begin{cases} \frac{x}{6} & , \ x = 1,2,3 \\ 0 & , \ other\ wise \end{cases}}$$

Find :

1_ Probability mass function

2_ cumulative probability function

Sol) 1-

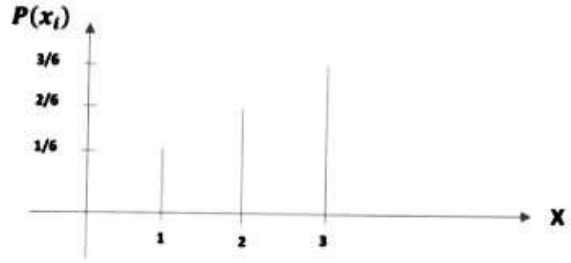
x	1	2	3
$P(X=x)$	1/6	2/6	3/6

$\sum P(X = x_i) = 1$

$$\sum P(X = x_i) = P(X = 1) + P(X = 2) + P(X = 3) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

تحقق الشرط

رسم الدالة



2_ $F(X) = P(X \le x)$

$F(0) = P(X \le 0) = P(X = 0) = 0$

$F(1) = P(X \le 1) = P(X = 0) + P(X = 1) = \frac{1}{6}$

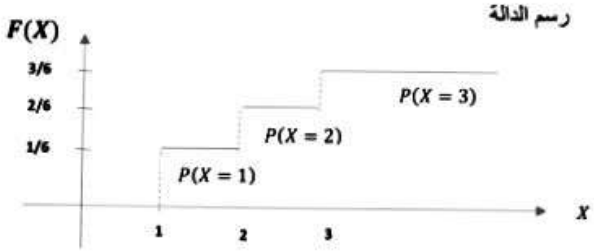
$F(2) = P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

$F(3) = P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

$$\therefore F(X) = \begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \le x < 2 \\ \frac{3}{6} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

رسم الدالة



وفي حالة توفر الدالة التراكمية فقط والمطلوب إيجاد دالة الكتلة الاحتمالية $P(X)$ يتم اجراء عملة الطرح من الأعلى الى الأسفل وفق الأسلوب الاتي :-

_ $P(0 < x \le 1)$

$$P(X = 1) = F(b) - F(a) = \frac{1}{6} - 0 = \frac{1}{6} \qquad P(a < x \le b)$$

_ $P(1 < x \le 2)$

$$P(X = 2) = F(2) - F(1) = \frac{3}{6} - \frac{1}{6} = \frac{2}{6}$$

_ $P(2 < x \le 3)$

$$P(X = 3) = F(3) - F(2) = 1 - \frac{3}{6} = \frac{3}{6}$$